# Hierarchical Incremental Slow Feature Analysis


M. Luciw, V. R. Kompella, and J. Schmidhuber

{matthew, varun, juergen}@idsia.ch

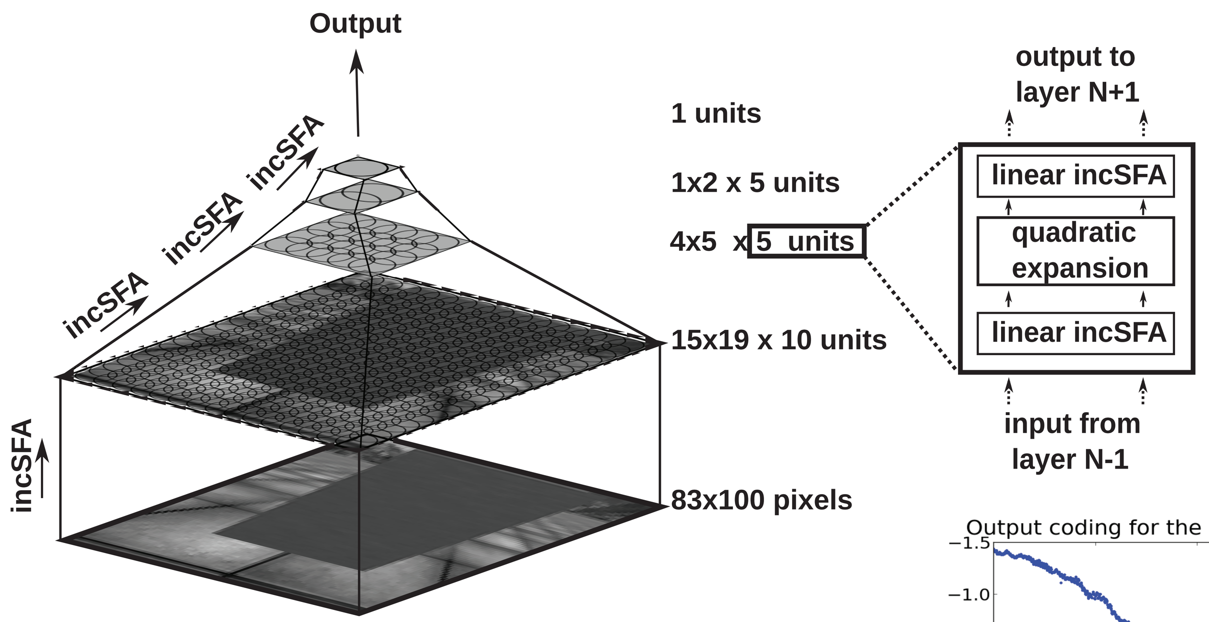
IDSIA-SUPSI-USI, Lugano, Switzerland

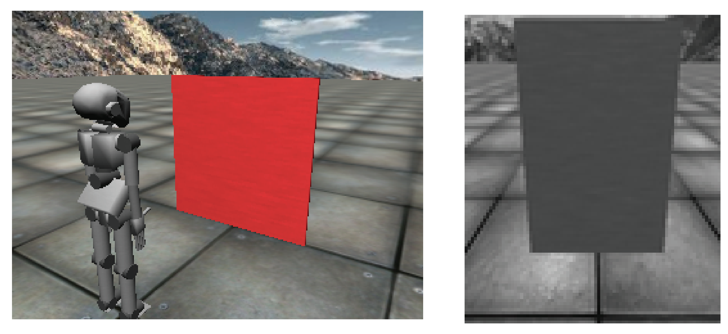

## Overview

- Slow feature analysis (SFA): an **unsupervised** learning technique for feature extraction from sequences driven by the *slowness* objective. Kompella *et al.* developed an Incremental SFA (IncSFA).
- IncSFA has linear time complexity w.r.t. input dimension, while Batch SFA (BSFA) is cubic. BSFA has quadratic space complexity, IncSFA is no worse than quadratic.
- IncSFA might suit autonomous agents: 1. advantageous computational complexity in the case of limited onboard hardware, and 2. it allows updating of existing slow features with new sensory input (i.e., if the environment changes) without any input storage.
- Hierarchical SFA (H-SFA) suits images. Via multiple layers of feature extraction within small receptive fields, it takes advantage of spatial locality to reduce dimension (and decrease the search space size), hopefully without losing key information about the properties of the world.
- We studied integrating IncSFA with H-SFA. Below are some preliminary results.

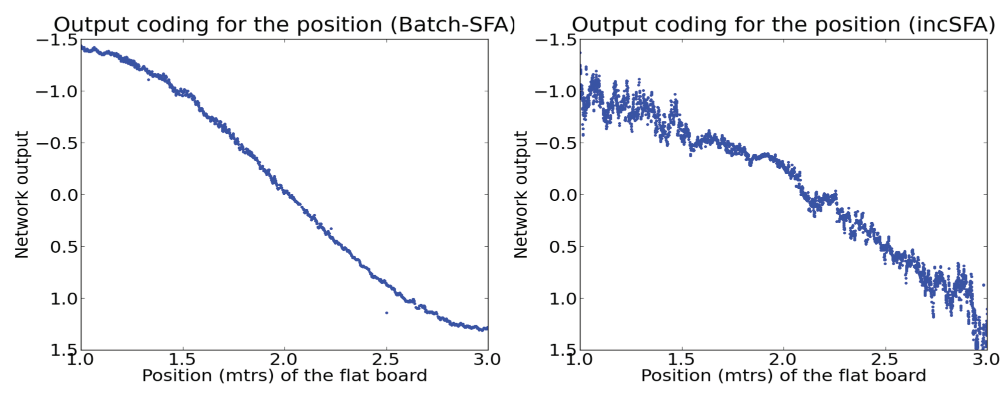
## H-IncSFA Results

**Proof Of Concept: All Layers Trained From Scratch with Incremental SFA**

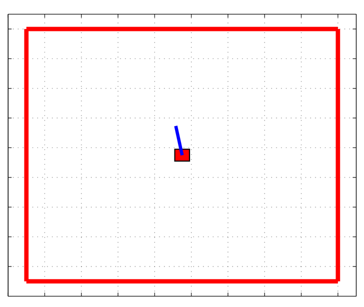


**Stable Camera, Moving Object (1D) Movement Paradigm *a la* Franzius**




**Much Harder Test:**



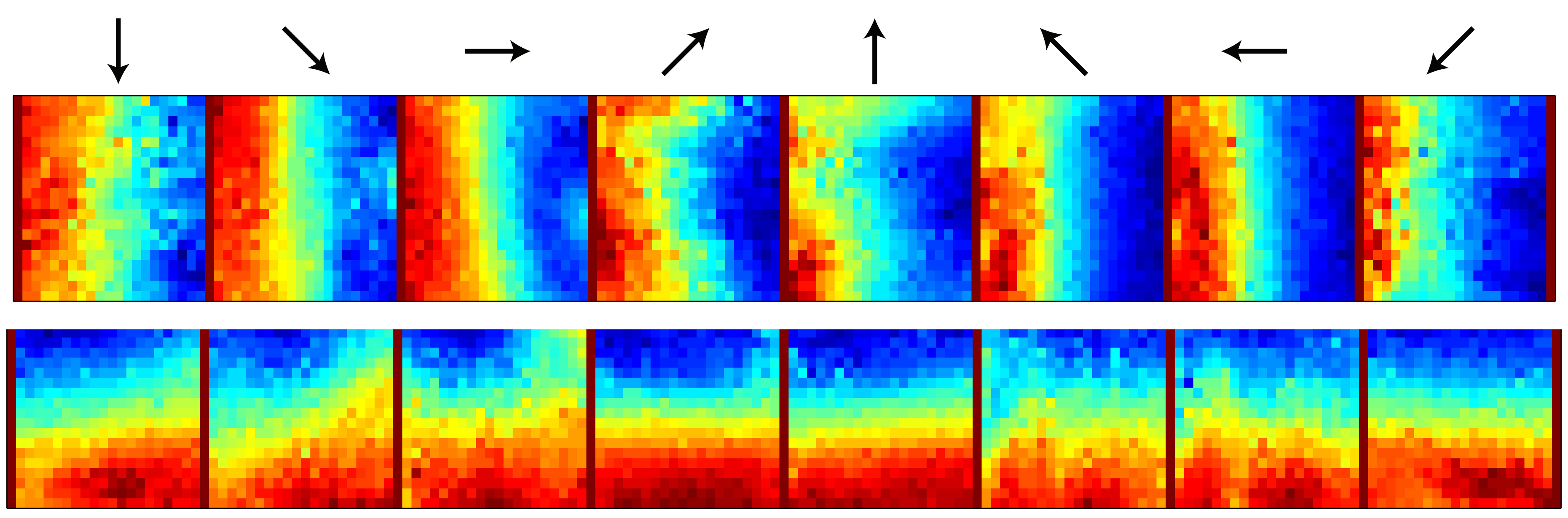
**Extract position from local views -Done using deep BSFA net by Franzius *et al.*, 2007**

**Movement Paradigm**
- High rotation velocity
- Low translation velocity
- Eventual uniform sampling in position and orientation

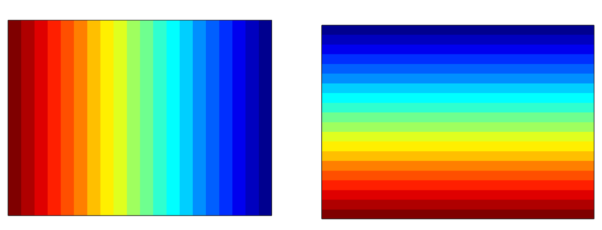
**Deep Net Parameters**

| | L1 | L2 | L3 | L4 |
|---|---|---|---|---|
| Input | 41x41x3 | 9x9x32 | 4x4x32 | 2x2x16 |
| RF Size | 7x7 | 3x3 | 3x3 | 2x2 |
| Num RFs | 81 | 16 | 4 | 1 |
| RF Offset | 4px | 2px | 1px | N/A |

**Network Training**
- The layers are trained in order, one at a time
- The highest layer uses Incremental SFA
- All layers will be able to adapt later via IncSFA

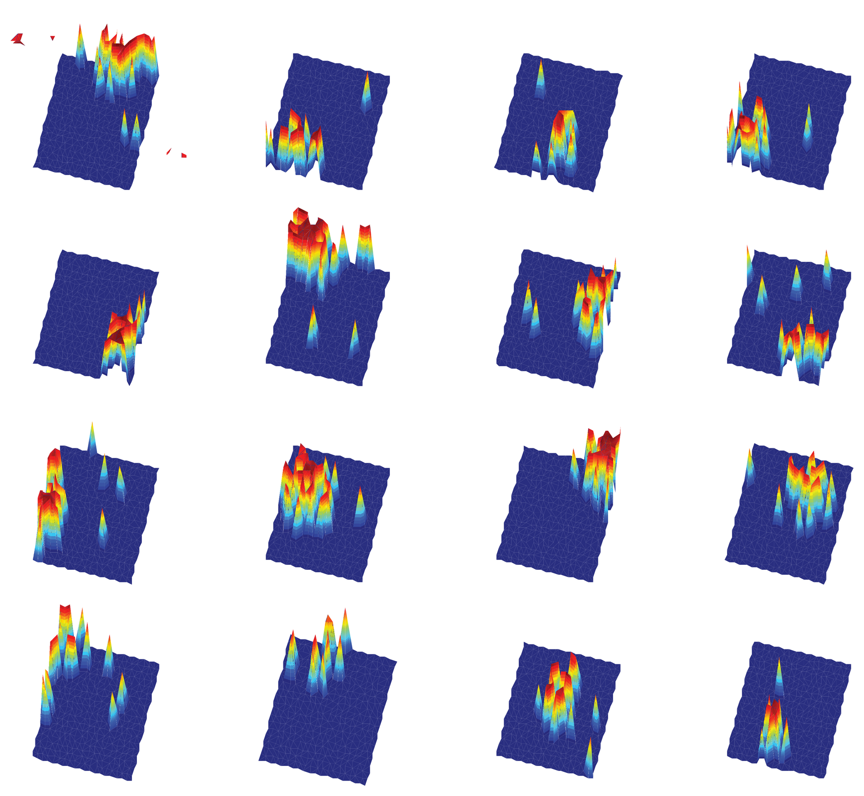


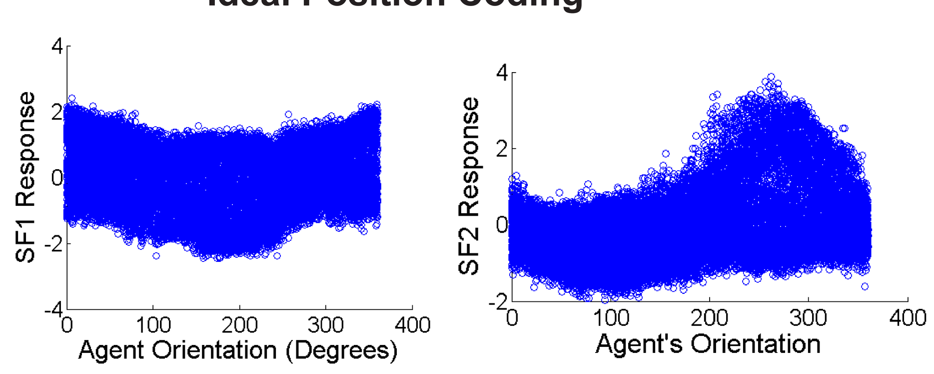
**Responses of Two Slowest Features wrt Position and Orientation (20,000 image sequence)**



**Ideal Position Coding**

**(Right): Layer 5. A competitive learning layer develops "place cells"**






**Mostly Orientation Invariant...**

## IncSFA Algorithm

**Algorithm 1:** IncSFA$(J, K, \theta)$

//Autonomously learn $J$ slow features from samples $\mathbf{x} \in \mathcal{R}^I$
// $\mathbf{V}$ : Matrix of $K \leq I$ principal component column vectors
// $\mathbf{W}$ : Matrix of $J \leq K$ slow feature column vectors
// $\mathbf{v}^\gamma$ : First PC in whitened difference space
// $\bar{\mathbf{x}}$ : Mean of $\mathbf{x}$
$\{\mathbf{V}, \mathbf{W}, \mathbf{v}^\gamma, \bar{\mathbf{x}}\} \leftarrow$ Initialize ()
**for** $t \leftarrow 1$ *to* $\infty$ **do**
  $\check{\mathbf{x}} \leftarrow$ Sense$(worldstate)$
  $\mathbf{x} \leftarrow$ Expand $(\check{\mathbf{x}})$ //non-linear expansion
  $\{\eta_t^{PCA}, \eta_t^{MCA}\} \leftarrow$ LearningRateSchedule $(\theta, t)$
  $\bar{\mathbf{x}} \leftarrow (1 - \eta_t^{PCA})\, \bar{\mathbf{x}} + \eta_t^{PCA}\, \mathbf{x}$ //Update mean
  $\mathbf{u} \leftarrow (\mathbf{x} - \bar{\mathbf{x}})$ //Centering
  //Candid Covariance-Free Incremental PCA
  $\mathbf{V} \leftarrow$ CCIPCA-Update $(\mathbf{V}, K, \mathbf{u}, \eta_t^{PCA})$
  $\mathbf{S} \leftarrow$ ConstructWhiteningMatrix $(\mathbf{V})$
  **If** $t > 1$ **then** $(\mathbf{z}_{prev} \leftarrow \mathbf{z}_{curr})$ //Store prev.
  $\mathbf{z}_{curr} \leftarrow \mathbf{S}^T\mathbf{u}$ //Whitening and dim. reduction
  **if** $t > 1$ **then**
    $\dot{\mathbf{z}} \leftarrow (\mathbf{z}_{curr} - \mathbf{z}_{prev})$ //Approx. derivative
    //To learn sequential addition parameter $\gamma$
    $\mathbf{v}^\gamma \leftarrow$ CCIPCA-Update $(\mathbf{v}^\gamma, 1, \dot{\mathbf{z}}, \eta_t^{PCA})$
    $\gamma \leftarrow \mathbf{v}^\gamma / \|\mathbf{v}^\gamma\|$
    //Covariance-free Incremental MCA
    $\mathbf{W} \leftarrow$ CIMCA-Update $(\mathbf{W}, J, \dot{\mathbf{z}}, \gamma, \eta_t^{MCA})$
  **end**
  $\mathbf{y} \leftarrow \mathbf{z}_{curr}^T \mathbf{W}$ //Slow feature output
**end**

**Algorithm 2:** CCIPCA-Update$(\mathbf{V}, K, \mathbf{u}, \eta)$

//Candid Covariance-Free Incremental PCA
$\mathbf{u}_1 \leftarrow \mathbf{u}$
**for** $i \leftarrow 1$ *to* $K$ **do**
  //Principal component update

$$\mathbf{v}_i \leftarrow (1-\eta)\, \mathbf{v}_i + \eta \left[\frac{\mathbf{u}_i \cdot \mathbf{v}_i}{\|\mathbf{v}_i\|}\, \mathbf{u}_i\right]$$

  //Residual

$$\mathbf{u}_{i+1} = \mathbf{u}_i - \left(\mathbf{u}_i^T \frac{\mathbf{v}_i}{\|\mathbf{v}_i\|}\right) \frac{\mathbf{v}_i}{\|\mathbf{v}_i\|}$$

**end**
**return V**

**Algorithm 3:** CIMCA-Update$(\mathbf{W}, J, \dot{\mathbf{z}}, \gamma, \eta)$

//Covariance-Free Incremental MCA
$\mathbf{l}_1 \leftarrow 0$
**for** $i \leftarrow 1$ *to* $J$ **do**
  //Minor component update
  $\mathbf{w}_i \leftarrow (1-\eta)\mathbf{w}_i - \eta\left[(\dot{\mathbf{z}} \cdot \mathbf{w}_i)\, \dot{\mathbf{z}} + \mathbf{l}_i\right]$.
  //Normalize
  $\mathbf{w}_i \leftarrow \mathbf{w}_i / \|\mathbf{w}_i\|$.
  //Lateral competition from "lower" components
  $\mathbf{l}_{i+1} \leftarrow \gamma \sum_j^i (\mathbf{w}_j \cdot \mathbf{w}_i)\mathbf{w}_j$
**end**
**return W**

## References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**www.idsia.ch/~luciw**
**www.idsia.ch/~kompella**
**www.idsia.ch/~juergen**